STANDARD GRADUATE MATHEMATICS COURSES, FALL 2025

The following classes offered in Fall 2025 are 3–unit graduate courses, not cross-listed to the undergraduate level. All of the following count as standard graduate courses for the PhD program.

Math 5043: Introduction to Numerical Analysis
TR 2:00-3:20
Prof. D. Szyld

This course provides a graduate level introduction to classical and modern methods for fundamental problems in computational science and engineering, including approximation and interpolation, numerical integration/quadrature, direct methods for systems of equations, and solution of systems of nonlinear equations. A rigorous mathematical approach to these topics is taken, including floating point arithmetic, error analysis, conditioning and stability, and convergence theorems. The course is accessible to graduate students from all areas of science and engineering interested in a mathematical foundation for the listed computational methods.

Textbook: J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Third Edition, Springer, 2010.

Math 8011: Abstract Algebra I
TR 9:30-10:50
Prof. Martin Lorenz

This course, the first semester of a year-long graduate-level introduction to abstract algebra, is roughly organized into three main parts: Groups (Chapters 1-6 of the textbook), Rings & Modules (Chapters 7-12), and Fields (Chapter 13). The indicated chapters from the textbook contain more material than can be covered in one semester; so a selection will be made. Also, the last part (fields) will likely not be completed in the fall semester and continue in the spring, when the main topic will be Galois Theory. My approach and conventions will occasionally differ from those adopted in the textbook, but I will post complete classnotes for my lectures shortly after each lecture so that a record of the material as covered in class will be available to students.

The abstract algebra sequence Math 8011/8012 is a prerequisite for many of the higher-level graduate courses in pure mathematics and it provides the background needed for the PhD qualifying exam in Algebra.

Prerequisites: Math 3098 or equivalent or permission of instructor.

Textbook: Dummit & Foote, *Abstract Algebra*, 3rd ed., John Wiley & Sons, 2004.

Math 8031: Probability Theory
MW 1:00-2:20
Prof. A. Yilmaz

In this course, we will introduce the axioms and fundamental notions of probability based on measure and integration, develop the theory with the accompanying probabilistic intuition, which is equally important, cover some of the important concepts and results regarding Markov chains and random walks, formulate and prove the strong law of large numbers for independent random variables, define and characterize weak convergence of probability measures, and give a rigorous treatment of the central limit theorem.

Prerequisites: You should be comfortable with undergraduate-level Real Analysis. Any previous exposure to undergraduate-level Probability Theory or graduate-level Real Analysis would be helpful, but the course will be self-contained in those regards, i.e., they are not prerequisites.

Textbook: L. B. Koralov and Y. G. Sinai, Theory of Probability and Random Processes, 2nd Ed., Springer, 2007 (corrected 2nd printing 2012). We will cover Chapters 1–10 of the book.

Math 8041: Real Analysis I
MW 9:00-10:20
Prof. C. Gutirrrez

This course is the first semester of a year-long sequence covering the core areas of analysis. It is especially intended for students in pure mathematics and in analysis-related fields such as partial differential equations, probability, functional analysis, and their applications.

Topics for the semester include: functions of bounded variation and the Riemann–Stieltjes integral; Lebesgue outer measure and Lebesgue measure; Lebesgue measurable functions and their properties; the Lebesgue integral and its fundamental theorems; comparison with the Riemann integral; Fubini's theorem and repeated integration; and differentiation of integrals. Emphasis will be placed on solving exercises and developing problem-solving techniques throughout the course.

Prerequisites: Math 5041 or a solid background in real variables and Euclidean topology, sequences of functions, and Riemann integration.

Textbook: Richard Wheeden and Antoni Zygmund, *Measure and Integral: An Introduction to Real Analysis* (Pure and Applied Mathematics) CRC Press; 2nd edition (2015), ISBN: 9781032918938.

Additional References:

- B. Makarov et al., *Selected problems in real analysis*, Translations of Math. Monographs, vol. 107, American Mathematical Society, 1992, ISBN: 0821809539. A rich collection of exercises and problems across various levels.
- E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration, and Hilbert Spaces* (Princeton Lectures in Analysis III) by Princeton University Press, 2005, ISBN: 0691113866.

Math 8061: Differential Geometry & Topology I
MW 10:30-11:50
Prof. M. Stover

This is a standard introductory graduate course in differential topology. Topics include: smooth structures on manifolds; smooth functions and maps between manifolds; tangent and cotangent bundles; vector bundles; differential forms; tensors; integration and Stokes' theorem.

Textbooks:

- Lee, *Introduction to Smooth Manifolds* (required)
- L. Tu, *An Introduction to Manifolds* (recommended)

Math 8107: Mathematical Modeling for Science, Engineering, and Industry
TR 9:30-10:50
Prof. B. Seibold

In this course, students form research groups for projects provided by partners in industry, engineering, or in other disciplines of science. Throughout the semester, the teams are advised by the course instructors and the external partners. The problems themselves are formulated in non-mathematical language, so the students are required to translate these into mathematical language and develop solutions using methods of mathematical modeling. This means that the mathematical and computational methods must be selected or created by the students themselves. At the end, research findings and final results need to be formulated in a language accessible to the external partner. Students disseminate their progress and achievements in weekly presentations, a mid-term and a final project report, as well as a final presentation. Group work with and without the instructors' involvement is a crucial component in this course. Graduate students from disciplines other than mathematics, interested in collaborating in teams on mathematical modeling, and with a background in principles of modeling and computational methods, are welcome to this course.

Textbook: None.

Math 8141: Partial Differential Equations I
TR 11:00-12:20
Prof. K. Morgan

Partial Differential Equations (PDEs) are used to describe a variety of phenomena and have applications in many fields such as physics, economics, neuroscience, and biology. This course provides a basic introduction to the mathematical theory of PDEs. We will study the qualitative and quantitative properties of solutions to three key equations: the Laplace equation, the heat equation, and the wave equation. This study provides the foundation for understanding the more general elliptic, parabolic, and hyperbolic equations.

The course will be useful for students in analysis, applied mathematics, geometry, physics, and engineering.

Prerequisites: Undergraduate real analysis; advanced calculus of several variables.

Textbook: L. C. Evans, *Partial Differential Equations*, Graduate Texts in Mathematics, vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.

Math 9014: Commutative Algebra
TR 12:30-1:50
Prof. V. Dolgushev

This course focuses on the fundamental concepts of commutative algebra. Topics of the course include ideals of commutative rings, modules, Noetherian and Artinian rings, Noether normalization, Hilbert's Nullstellensatz, rings of fractions, primary decomposition, discrete valuation rings and the rudiments of dimension theory.

Textbook: Atiyah and MacDonald, *Introduction to Commutative Algebra*

Math 9061: Lie Groups
TR 2:00–3:20
Prof. M. Stover

This course develops Lie theory from the ground up. Starting with basic definitions of Lie groups and Lie algebras, the course develops structure theory, analytic and algebraic aspects, structure of nilpotent and solvable Lie groups, and the classification of semisimple Lie groups.

Textbook: None.